A Folding Horizon Approach for Setting Gates of Activities in the Stochastic Project Scheduling Problem

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1 Introduction

In stochastic models where decisions need to be made prior to observing the entire set of realizations, a way to reduce the uncertainty induced by the stochastic elements is by introducing flexibility into the decision variables and allowing changes on these decisions over time, after some of the stochastic elements are realized. Introducing flexibility has become quite common in Supply Chain Management. In [6] and [7] flexibility is introduced in contracts with suppliers through the “commitment time frame”, i.e. the decision of when to order. In [1], retailers commit to purchase quantities within pre-defined intervals. A new approach is introduced in order to adjust decisions dynamically — “a folding horizon approach”. Folding horizon approaches are also used in the area of Revenue Management (see [12]) especially regarding pricing decisions. The aim of this paper is to implement a folding horizon approach in the field of Project Management. Adjusting future decisions in a dynamic way could help the project manager (PM) to reduce uncertainty, thus to reduce his costs. More precisely, if an activity is supposed to start its processing in the distant future, it would be logical to determine for it a “nominal” predetermined start time, and then to change this decision when more information is obtained. It would also be logical not to perform changes at a time close to the nominal one. In the literature it is outlined as setting a “frozen zone” before each decision (see [6]). Of course in reality there is a penalty cost for changing the terms of the contract. There is thus a trade-off between the penalty costs for changing the nominal gates and the potential savings induced by the reduction of the uncertainty.

Scheduling of activities with stochastic activity durations has been investigated extensively ever since the introduction of the Program Evaluation and Review Technique (PERT, see [5, 8]). Most of the existing models share the same objective — makespan minimization. In recent years, new methodologies have evolved that focus on another objective — maximizing the net present value (NPV) of projects (see [9]). Maximization of NPV in stochastic settings was considered, to the best of our knowledge, only by Buss and Rosenblatt [3] and by Sobel et al. [11]. In a recent paper ([2]) we proposed a new approach to project scheduling with uncertain activity durations. This approach consists of determining for each activity a gate, a time before which the activity cannot begin. This approach was motivated by models proposed in [4] in the context of machine scheduling and in [13] and [14] in the context of project scheduling.

We consider the following environment: a project is composed of activities that are processed by subcontractors, or that require that the resources must be booked and be ready for the activity at a predetermined time. The activities have durations that are characterized as random variables. The project has an external due-date and there is a tardiness penalty for each unit of time in which the project is delayed beyond that due-date. The PM seeks to determine for each activity a gate — a time before which the activity cannot begin. Once they are determined, the processing of an activity cannot start before its gate but it may start later. The PM is confronted therefore with the following cost structure: (1) “holding costs” that are incurred in the case where an activity is ready to start (as its predecessors are done) but cannot actually start because the resources required for it were planned to arrive at a later time. (2) “shortage costs” that are incurred in the case where the resources required for an activity are ready to be used but the activity cannot start because of precedence constraints.

In [2], all the gates were determined in a “static” way, before any uncertain parameters (the durations) are realized. In this way, all the risk induced by the uncertainty is assumed by the PM only. Eventual changes of these decisions can be achieved in two scenarios: (1) there is a maximal time for each activity to perform a change on its nominal gate, and a cost per time unit for performing this change. (2) We consider potential changes at specific predetermined times. In each of these potential times, we decide whether or not to perform changes on gates of any of the future activities. There is a cost per time unit for performing a change in the gate of an
activity; in addition this cost can vary according to the time we perform the change. The goal of this paper is to introduce, given a specific scenario, a folding horizon approach to the problem of setting the gates of the activities so as to minimize the expected penalty and changes costs.

2 Employing CE method to the dynamic problem of setting gates

The CE method was developed by Rubinstein and Kroese [10]. It is a general heuristic method for solving estimation and optimization problems. For optimization problems, the CE method requires the following two phases: (1) generation of a sample of random data according to a specified random mechanism, and simultaneous calculation of the objective function values; (2) updating parameters of the random mechanism (on the basis of the data collected) in order to produce an improved sample in the next iteration, that will improve the value of the objective function. To apply the CE method to our problem, we used the following steps: first, we solve the algorithm developed in [2] to compute the nominal vector of gates. Then, we move to the next time where changes are allowed. At each of these times, we start a CE procedure as in [2], with some changes: in each iteration of the CE procedure, we generate new possible solutions just for the following activities: in scenario 1, for one activity at a time, according to its maximal time; in scenario 2, for all activities that still did not begin their processing.

3 Computational Study

To get some insights on the benefits of a folding horizon approach, we also consider a “no-cost” scenario. Although unrealistic, it is used for comparison purposes only. In this scenario, we consider changes at predetermined times as in scenario 2, but changes can be performed without incurring any cost. We study the performance of the folding horizon approach in the three scenarios. All the algorithms were programmed in Matlab and applied to three sets of four random projects with a random number of activities between 5 to 20, 20 to 30, and 30 to 40, respectively. We compare the performances of each of the three scenarios to the algorithm developed in [2], and compute the percentage of amelioration in the cost. For each generated project, we run the algorithms four times (with different realizations). The results that were obtained are summarized in Table 1. The percentage presented is the average of four replications for each one of four projects from a specific size.

<table>
<thead>
<tr>
<th>Project size</th>
<th>Percentage of amelioration</th>
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<tbody>
<tr>
<td></td>
<td>“no-cost” scenario scenario 1 scenario 2</td>
</tr>
<tr>
<td>5–20</td>
<td>10.9823 7.1025 5.3587</td>
</tr>
<tr>
<td>20–30</td>
<td>14.5095 7.3420 5.8118</td>
</tr>
<tr>
<td>30–40</td>
<td>16.1889 7.0378 5.1341</td>
</tr>
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Table 1: Percentage of amelioration of the folding horizon approach for different sizes of projects.

Table 1 shows that in the utopic “no-cost” scenario, the percentage of amelioration grows as the size of the project grows. This phenomenon disappears in scenarios 1 and 2. The explanation is that in the “no-cost” scenario, as the size of the project grows, we save more penalty costs since the project involves more activities but also since it may be longer such that we have more opportunities to adjust the gates according to past information. In scenarios 1 and 2, as the size of the project grows, we save more penalty costs but since there are more opportunities to perform changes, the cost of changing will also grow, leading to the vanishing of the phenomenon of “benefit of scale”. From Table 1 we see that the percentages of amelioration are the highest in the “no-cost” scenario, since we do not have any cost for performing changes. The percentages in
scenario 1 are higher than these of scenario 2: this can be explained by the fact that in scenario 2, we can perform changes to more activities in parallel.

4 Concluding remarks

In this paper, we developed a methodology for adjusting tasks release decisions in a folding horizon way. We use the static algorithm to determine at time zero a basic guideline for the contracts with the subcontractors. Then, according to updated information on precedent activities and the scenario, we check if it is worthwhile to perform a change. The results obtained show that implementing a folding horizon approach considerably lower costs since it allows a deferment of decision making. Implementing this method also allows the subcontractor to share uncertainty with the PM. The CE algorithms based on the folding horizon approach can be used in a large variety of OM problems. For example, it can be used in the Revenue Management field, regarding pricing issues or in the field of Supply Chain, especially for products with a high level of uncertainty in the demand, like fashion items and seasonal products.

References


