On the price of demand censoring in the newsvendor problem

Omar Besbes*  
Columbia University

Alp Muharremoglu†  
Columbia University

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Abstract

We consider a repeated newsvendor problem where the decision-maker (DM) does not have access to the underlying distribution of discrete demand. We analyze three informational settings: i.) the DM observes realized demand in each period; ii.) the DM only observes realized sales; and iii.) the DM observes realized sales but also a lost sales indicator that records whether demand was censored or not. We provide a characterization of the best achievable performance in each of these cases, where we measure performance in terms of regret: the worst case difference between the cumulative costs of any policy and the optimal cumulative costs with knowledge of the demand distribution. In particular, we show that for both the first and the third settings, the best achievable performance is bounded (i.e., does not scale with the number of periods) while in the second setting, it grows logarithmically with the number of periods. The results enable one to quantify the performance degradation stemming from demand censoring and identifies minimal information through the lost sales indicator that mitigates some of this degradation.

Keywords: inventory management, newsvendor, estimation, nonparametric, demand censoring

One of the major factors driving firms to hold inventories is the uncertainty in customer demand realization in conjunction with the inflexibility of supply sources. Consequently, a key input in any stochastic inventory model is a description of the uncertainty characterizing demand realizations, most often given in the form of a probability distribution. This input, together with cost parameters allows the decision-maker to trade-off different cost sources such as, e.g., holding costs, lost revenue and loss of customer good-will through inventory unavailability.

In practice, such a demand distribution would typically need to be estimated from historical data. A first question that arises pertains to the magnitude of the loss in performance due to not knowing the demand distribution and having to rely only on past observations. However, one should note that in many instances, the estimation of a demand distribution and inventory management decisions are closely tied as the decisions might influence the data that one collects. For example

*Graduate School of Business, e-mail: ob2105@columbia.edu
†Graduate School of Business, e-mail: alp2101@columbia.edu, contact author
in retail, it is common for the demand that exceeds the available inventory to be lost, and in most cases this excess demand is not observable by the firm. Therefore, the firm may only have records of past sales, as opposed to actual demand. This phenomenon, commonly referred to as demand censoring, inevitably comes at a cost to the firm, and the purpose of this paper is to provide a characterization of its impact on the cost of the firm, and guidance on the policies that a firm may follow.

To that end, we study a classical inventory system, a repeated newsvendor problem. For such a problem, it is well known that, given a demand distribution, a critical fractile solution is optimal. We assume that inventory and demand are discrete and that the demand distribution is unknown to the decision-maker and s/he has only access to data s/he collects over time. We measure the performance of a policy in terms of regret, where for a given policy its regret is the difference between its expected cumulative cost over a given period of time and the optimal expected cost one could have achieved had one known the distribution of demand in advance. To ensure a robust objective, we aim to characterize the best possible worst-case regret (minimax regret), where the worst-case will be taken over a broad class of distribution functions.

In order to characterize the effects of censoring, we will focus on different informational settings. We first analyze a benchmark case where demand is observable and hence no censoring is present. As time progresses, the decision-maker accumulates demand observations and can use these to determine her/his inventory decisions. We then focus on the case where only sales are observable, and only those can be used for decision-making; this is the censored demand case. The difference between the regret in both settings can be interpreted as the price in performance that one pays due to censoring. In addition, we will also study an intermediate informational setting we call “partial censoring” where the decision-maker, in addition to observing sales, also observes whether demand exceeded (strictly) sales or not, i.e., observes whether any sales were lost. The motivation for this third setting is dual. First, the lost sales indicator could be seen as the minimal information one could collect beyond sales. Second, the availability of the lost sales indicator has been assumed, to the best of our knowledge, in all studies that have appeared in the literature and that analyze newsvendor problems with an unknown discrete demand distribution (and discrete inventory levels); hence, the analysis of this setting enables one to relate to previous studies. The literature review below discusses further this point.

The main contributions of the paper can be summarized as follows.

i.) We establish that when the decision-maker has access to demand observations, s/he can achieve a bounded worst-case regret. In other words, the worst-case regret will not grow beyond a certain value as the number of periods increases.

ii.) We provide a fundamental lower bound on worst-case regret when demand is censored. In particular, we establish that any policy will have to incur a worst-case regret of order \(\log T\),
where \( T \) is number of periods.

iii.) We provide a policy that relies only on censored demand and that nearly achieves the lower bound above.

iv.) We establish that the availability of the lost sales indicator, in addition to censored demand, enables a decision-maker to recover a bounded worst-case regret.

In addition, we illustrate through numerical experiments the performance of the proposed algorithms in the various settings studied.

To highlight the key differences between the settings analyzed through a simple example, suppose for a moment that the inventory level in a given period is 25 and that demand turns out to be 40. Then sales are equal to 25. In the uncensored case, the decision-maker would observe the value of demand, 40. In the censored case, the decision-maker would only know that the demand was greater than or equal to 25, but would not be able to observe whether demand was equal to 25 or strictly greater than 25. In contrast, in the partially censored case, the decision-maker would know that demand was greater than 25.

At a qualitative level, the key intuition underlying the degradation in performance due to censoring is that one cannot refine one’s confidence about the optimality of a given inventory level while using such a level. To see this, note that the optimal inventory decision, \( x^* \), is the minimum level such that \( \mathbb{P}(\text{Demand} \leq x^*) \geq \beta \), where \( \beta \) is the so-called critical ratio. Suppose one orders \( \hat{x} \), one’s current estimate of \( x^* \), for a number of periods. For those periods, due to censoring, the inventory manager observes whether demand was greater than or equal to \( \hat{x} \) (which is equivalent to observing whether demand was less or equal than \( \hat{x} - 1 \)) but not whether demand was equal to \( \hat{x} \), and hence cannot use these observations to reliably refine its estimate of \( \mathbb{P}(\text{Demand} \leq \hat{x}) \). However, given the definition of \( x^* \), refining one’s estimate of \( \mathbb{P}(\text{Demand} \leq \hat{x}) \) is needed in general in order to improve the confidence in one’s estimate of \( x^* \). As a result, in the presence of censoring, the decision-maker will need to periodically step away from an ordering quantity that seems optimal to be able to refine her/his estimate as time progresses. In essence, the policy we design in the censored case actively balances exploration and exploitation, and steps away from a current best estimate of an optimal ordering quantity at a frequency of roughly \((\log T)/T \). In the partial censoring case, however, one can fine-tune the estimate for the optimal ordering quantity without active exploration, removing some of the tension between exploration and exploitation.

Overall, the analysis outlined in the paper enables one, through both lower and upper bounds on performance, to understand some of the critical issues associated with censoring, how to counter them effectively, and what minimal information would mitigate censoring effects on performance. These results are novel and complement well the existing literature on inventory models with unknown demand distribution (with or without censoring).