Abstract

We consider a general infinite horizon inventory control model which combines demand and supply risks and the firm’s ability to mitigate the supply risks by diversifying its procurement orders among a set of $N$ potential suppliers. Supply risks arise because only a random percentage of any given replenishment order is delivered as useable units. The suppliers are characterized by the price they charge and the distribution of their yield factor. Assuming unsatisfied demand is backlogged, the firm incurs, as in standard inventory models, three types of costs: (i) procurement costs; (ii) inventory carrying costs for units carried over from one period to the next and (iii) backlogging costs. We establish the existence of an optimal stationary policy, under both the long-run discounted and average cost criterion, and characterize its structure. Assuming each period’s inventory level distribution can be approximated as an Normal, we develop an efficient solution method identifying additional structural properties. Finally, we identify a simple class of heuristic policies which come close to being optimal.
We consider a general infinite horizon inventory control model which combines demand and supply risks and the firm’s ability to mitigate the supply risks by diversifying its procurement orders among a set of $N$ potential suppliers. Supply risks arise because only a random percentage of any given replenishment order is delivered as useable units that meet all quality standards and specifications. This percentage is referred to as the order’s yield factor, with a supplier specific distribution. This general yield model can be used to represent the risks of a complete supply disruption, due to sabotage, labor strikes, terrorist attacks, natural disasters, health hazards, etc., as well as partial yield losses below a benchmark standard. The former type of supply risk may be modeled by specifying that the distribution of the supplier’s yield factor, the so-called yield distribution, has a positive mass in zero. Demand risks consist of each period’s demand volume being unknown, at the start of the period, as in classical inventory models.

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This paper is a companion to Federgruen and Yang (2009) which analyzed the finite horizon version of this model and derived many structural properties of the optimal procurement strategy: for example, while the aggregate order policy fails to be of a base-stock type, there exists a threshold inventory level, in each period, such that it is optimal to place orders if and only if the period’s starting inventory is below this threshold. The optimal set of suppliers is consecutive in the suppliers’ effective cost rates, defined as the ratio of the supplier’s unit price divided by his expected yield factor. The optimal number of retained suppliers decreases with the starting inventory level, while the set of suppliers consists of those whose effective cost rate falls below a given benchmark cost rate and the optimal order vector can be determined as the unique solution of at most $N$ systems of $N$ equations in $N$ unknowns.

In this paper, we extend all of these structural properties to infinite horizon settings, both under the discounted and long-run average cost criterion. As is well known, the extension to, in particular, the long-run average cost criterion is very challenging, as the underlying Markov Decision Process has both a continuous unbounded state space and continuous feasible action sets.

As with almost all analyses of the long-run average cost criterion, ours proceeds by characterizing the asymptotic behavior of the optimal long-run discounted cost function, as the discount factor approaches one; see Sennott (1999) and Hernández-Lerma and Lasserre (1996, 1999). A key property to be established is that the relative difference between the long-run discounted optimal cost in any given starting state and that in some reference state, is bounded in the discounted factor, i.e., even as the absolute value of the optimal long-run discounted cost goes to infinity. Key
to most proof techniques for this boundedness property is to establish that the trajectory from any given starting state and that from the reference state can be made to coincide after a finite time interval. Under random yields with arbitrary continuous or mixed distributions, the above pairing of the state trajectories is considerably more challenging than when a given inventory level can be targeted with probability one by the choice of an appropriate size order. In addition, we obtain not only that a stationary optimal policy exists but that such a policy can be found as one satisfying a solution to the so-called long-run average optimality equation; this allows us for a straightforward extension of the various structural properties obtained for the discounted model. Note that in many models, for example, Feinberg and Lewis (2007) and Schäl (1993), at best, the existence of a solution to optimality inequalities can be obtained, a considerably weaker optimality result.

As mentioned, our structural results allow us to compute the optimal order vector in any given period as the unique solution of at most $N$ systems of $N$ equations in $N$ unknowns. However, even the evaluation of these equations for a given order vector is numerically complex, for general yield and demand distributions, in particular when the number of suppliers is moderate to large. We therefore develop a considerably more efficient solution method, applicable both in the finite and infinite horizon cases, which applies as an exact method when all distributions are Normal and as a close approximation for general distributions. This approach has the additional advantage of providing insights into the optimal scheme to allocate aggregate orders among the potential suppliers. In particular, we show that a supplier’s market share is given by the relative value of a specific supplier score, which is the product of a reliability and a cost score: the former is the mean-to-variance ratio of the supplier’s yield distribution, and the latter is given by the amount by which the supplier’s effective cost rate falls below the above mentioned benchmark cost rate. The market share of each selected supplier is given by his overall score relative to the sum of the suppliers’ scores. In particular, as with the supplier set itself, the market shares in any given period depend on all future costs, yield and demand distributions only via a single measure, i. e., the aforementioned benchmark cost rate.

While this solution method is considerably more efficient than the general exact method, it still requires several hours of CPU time to achieve an optimality gap of less than 1%, say, on a standard state-of-the-art platform. (See §5 for details.) In addition, it is somewhat challenging to implement the generated policy, in that the manager is provided with an algorithmic oracle rather than a simple, intuitive policy rule.

To address both complications, we propose a class of so-called linear inflation heuristics. Here, the aggregate order, in any period, is determined as the shortfall with respect to a given base stock level, inflated with a so-called inflation factor to account for the random yield losses. As to the allocation of these aggregate orders, the heuristic policy adopts constant allocation percentages, determined by the above mentioned market share formula, however with a single constant benchmark cost rate. (The exact benchmark cost rate depends on the starting inventory.) A policy within this heuristic class is specified by three parameters: the base stock level, the inflation factor.
and the benchmark cost rate. We derive a closed form approximate expression for the long-run average costs under any policy in this class and show how the parameter triple which minimizes this cost expression can be obtained by minimizing a single closed form function of one variable. A simulation study verifies that the approximate closed form long-run average cost expression is very accurate and generates a heuristic policy which is close to optimal.

References


