Optimal Inventory Control in Market-Making with Risk Aversion

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Market-makers have the obligation to trade fixed amounts of assets at quoted bid or ask prices, and their inventories are exposed to the potential loss when the market price moves in an undesirable direction. One approach to reduce the risk associated with price uncertainty is to adjust the inventory at the price of losing potential spread gain. For a single-asset model, we show that a threshold inventory control policy is optimal for mean-variance analysis and exponential utility criterion. The mean-variance analysis for a multiple-asset model suggests that there exists a simply connected no-trade region and that the optimal strategy can be obtained from the no-trade region.
Investors trade foreign currencies, securities and other financial products frequently. Unfortunately, there is no guarantee that every investor who wishes to buy (or sell) a certain amount of asset will find a counterparty willing to sell (or buy) the same amount at that time. This is exactly the objective of the so-called “market-makers”: to facilitate the trading process for most financial products. That is, the market-maker is ready to assume the role of a counterparty when one wishes to buy or sell financial products.

Typically, market-makers quote a pair of bid/ask prices to clients and have the obligation to buy/sell at the quoted prices if their clients wish to deal at the quoted price. Over time, market-makers buy at bid price and sell at ask price, which is higher than the bid price at any given instant. Their objective is to profit from the “spread” between bid and ask prices, not from price movements. In that regard, they are different from ordinary investors, who seek to profit by betting on price moves.

Market-makers encounter difficulties when receiving consecutive trades in the same direction. For example, suppose a market-maker holds no foreign currency initially and receives a series of sell orders afterwards (i.e., the clients sell to the market-maker), the market-maker’s holding position becomes very large and positive. This is potentially very risky because if the foreign currency depreciates, the market-maker will lose a considerable amount. For a risk-averse market-maker, this is certainly undesirable. Thus, he cannot simply wait for the arrival of a client (who wishes to buy the foreign currency from him) and sell to this client to bring his holding position back to zero. To reduce the risk and avoid such situations, the market-maker may consider selling certain amount of the foreign currency to other market-makers to lower his position instead of waiting for sell orders. When he sells to other market-makers, he becomes their client and has to sell at others’ bid prices. As a result, he forgoes the possibility of selling to his own clients at the ask price and taking the spread. Here we have a typical trade-off between profit and risk. Our goal in this research is to apply dynamic programming techniques in order to investigate when and by how much a market-maker should sacrifice profit to reduce risk.

The observation that market-makers may carry unwanted inventories has long caught the attention of the research community, and most previous work investigates how inventories influence the market-makers' behavior when quoting bid and ask prices. The theoretical analysis in Ho and Stoll (1981) shows that risk-averse market-makers will actively induce movements toward a desirable inventory level by setting favorable bid/ask prices. Empirical studies identified the impact of inventory on pricing, but that the evidence of inventory effects is rather weak compared with other components, for example, asymmetric information.

A survey of US foreign exchange traders (c.f. Cheung and Chinn 2001) indicates that the market norm is an important determinant of the bid-ask spread and only a small proportion of
bid-ask spreads differ from the conventional spread. Only 2% of respondents say inventory-cost related factors have impact on bid-ask spreads. This is because quoting volatile bid/ask spread may damage the market-maker’s reputation and drive away potential trading opportunities. In the meantime, the market-makers are reluctant to reveal adverse positions by quoting non-conventional spread. These results imply that in the foreign exchange market, the inventory is not managed by quoting bid/ask prices and it is controlled by trading with other market-makers instead. In fact, the trading volume is extremely high in the foreign exchange market. Passing unwanted inventory from one market-maker to another is commonly believed to be an important contributor to the trading volume (c.f. Lyons 2001). The survey of Cheung and Chinn (2001) also states that more than half of respondents believe that large players dominate dollar-pound and dollar-Swiss franc markets. Therefore, many small and medium-sized players have no market power, i.e., they have no impact on future price movements when actively trading with other market-makers.

Thus, our objective is to identify effective strategies for an inventory manager (i.e., market-maker) who does not control prices and can merely adjust inventory through active trading. In this sense, the market-making problem shares some important features with the classical inventory control problem. We need to determine the amount of assets to buy or sell during market-making process, which is analogous to the ordering quantity in inventory control. Indeed, in our case, the risk induced by inventory is analogous to the inventory holding cost, and the sacrificed spread profit due to active trading plays a similar role to the linear ordering cost. The sacrificed spread profit is the loss of spread encountered by a market-maker who sells/buys a unit of inventory to other market-makers (at their own price) rather than holding that unit of inventory and profiting from the spread in the future. Of course, there are some important differences: in the classical inventory control model, the order quantity must be non-negative and the unit inventory holding cost is deterministic, which as we shall see, are essentially different from the market-making situation.

To present our contribution, we need to define a threshold policy. Such a policy is defined by two parameters, an upper limit and a lower limit. Whenever the inventory is higher (lower) than the upper (lower) limit, the market-maker will decrease (increase) the inventory to the upper (lower) limit. Otherwise, i.e., when inventory level is between the two limits, the market-maker will not change its position. A symmetric threshold policy is one in which the upper (lower) limit is non-negative (non-positive) and the two limits are equal in absolute values.

- We propose the dynamic programming model for the market-maker inventory control problem, which considers the risk-averse attitude of the market-maker.
- We show that a threshold policy is optimal when the market-maker manages a single asset.

We also establish the optimality of a symmetric threshold policy as well as the monotonicity
of the symmetric threshold level under certain conditions.

- When the market-maker manages multiple assets simultaneously, there exists a no-trade region which is connected without holes. We identify important properties of the no-trade region and show that the optimal policy is obtained directly from the no-trade region. In addition, the no-trade region is symmetric with respect to 0 under certain conditions.

Based on these structural properties of the optimal policy, we could obtain efficient algorithms to solve the dynamic programming problems and the computational complexity is linear in the number of periods.

References

