Dynamic Assortment Planning

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Abstract

We investigate optimal assortment planning strategies for a retailer with limited shelf space. The retailer can choose among basic and fashion items with low and high risk (and return) respectively. Our motivation is in the apparel industry, and we explicitly model the vogue as a stochastic process that the retailer tries to follow. The objective is to maximize the long-term value of the retail business by dynamically adjusting the menu of products on display.

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1. Brief description of the research problem

Most of the assortment literature assumes that future preferences are stable (Kahn and Lehmann 1991). In that case, the chief reason to offer variety in a retail offering is to appeal to the heterogeneity of tastes in the target population. The stability assumption naturally leads to a static assortment policy. However, stable preferences should not be taken for granted. Indeed, March (1978) points out that attitudes about possible outcomes in the future are not entirely predictable and the variance in the subjective probability distribution over possible future preferences increases as the time horizon is lengthened. This is even more relevant for products with a strong fashion component.

In this paper we challenge the stability assumption and instead allow future preferences to vary. For that purpose, we consider a retailer managing a specific category of products that she sells to a market of consumers whose preferences change continuously over time. The retailer's problem is to dynamically select the assortment of products to offer based on her evolving forecast of consumers' demand and preferences. The retailer's objective is to maximize the infinite-horizon expected discounted profits generated by her retail business which include the revenues obtained by serving the demand and the costs of implementing a dynamic assortment strategy.

2. Key methodology and assumptions

The retailer's problem is to dynamically select the assortment to sell in order to maximize her expected discounted payoff over an infinite horizon. The following are specific characteristics of our model.

**Assortment:** Products within the category are differentiated by a single attribute taking values in a given set $S$. For example, depending on the category, this attribute could be color (for clothes), flavor (for food groceries), speed (for microprocessors), or some aggregated attribute that combines multiple characteristics of the product. For simplicity, we assume that the attribute takes numerical values.

**Demand:** We model market demand using a one-dimensional location model as in Hotelling (1929) and Lancaster (1966, 1975), but similar to Salop (1979) we consider an infinite line or circle to avoid 'corner' difficulties. According to this demand model, every consumer has a most preferred reference in $S$ that he would like to buy. To capture heterogeneity in consumers' preferences, we assume that the distribution of this most preferred reference across the population of consumers at a particular time is given by a probability distribution function, which depends on the value of a particular stochastic process known as the *vogue process* (described below). This feature of our model is well-suited for those categories of products for which the notion of fashion or market trend has a significant effect on demand.

**Vogue Process:** As mentioned before, a distinguishing feature of our model with respect to the existing literature in assortment planning is that we allow consumer preferences to change dynamically over time. To model this feature, we introduce a real-valued stochastic process $V_t$ which we interpret as the most popular reference across the entire population of consumers at a given point in time $t$. Hence, the movement of $V_t$ captures the evolution of the preferences of the consumers. We restrict our attention to vogue processes that evolve as one-dimensional diffusion processes. This choice implies that $V_t$ has (a.s.) continuous path so customers' preferences exhibit serial correlation. In other words, two consumers arriving close to each other will have similar preferences in a statistical sense.

**Replenishment:** Similar to Caro and Gallien (2007), we assume a perfect inventory replenishment process so that there are no stockouts or lost sales (i.e., realized sales equal total demand). On the other hand, we assume that changing the assortment is a costly process that incurs a fixed cost $K$. We also
assume that there is a lead time of \( T \), which represents the time it takes the production system and supply chain to deliver the new assortment.

**Pricing:** To focus on the assortment problem we assume that all products in the category have the same price.

For tractability reason, we consider the special case in which the vogue process \( V_t \) is a martingale; specifically a drift-less Brownian motion with constant volatility. This represents the case in which the seller cannot anticipate the instantaneous evolution of the vogue process. We then find convenient to introduce the gap process \( Z_t \) which serves as an auxiliary state variable that measures the distance between the vogue and the actual assortment. Note that the seller's problem is an impulse control problem (e.g., Harrison et al., 1983, Cadenillas and Zapatero, 1999) as her actions correspond to jumps in the evolution of the gap process every time she changes the assortment. However, due to the production lead time, there is an important difference between the seller's problem and the standard impulse control problem since there is a lag between the time a choice is made and the time the jump actually occurs.

3. Summary of major results

When the vogue process is a martingale the retailer's assortment problem can be formulated as an optimal stopping problem where the optimal policy is to change the assortment whenever the gap process \( Z_t \) moves beyond a threshold band. We show that the threshold can be determined by finding the unique solution to a fixed-point equation. We use this to derive insight on original assortment problem. In particular, we study when a static assortment is optimal (i.e., when it is not worth to follow the trend). We also study the trade-off between the switching cost \( K \) and the lead time \( T \) which we use a proxy to model different type of supply chains (such as efficient or agile supply chains, see Fisher 1997 and Lee 2004). In this regard, our results shed some light on how different supply chain characteristics favor different assortment planning strategies. We provide an extension of our model to the case when the vogue's state space is restricted to a circle. This allows us to capture the circular pattern which is sometimes observed in practice.

**References**


