Outsourcing and Backsourcing under Demand Uncertainty: A Simplified Real Options Model

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This paper examines the value that firms create by building outsourcing and backsourcing flexibility while operating under demand uncertainty. Unlike earlier studies, it provides an analytical investigation regarding the probability of switching to the outsourcing option and the expected time to make a switch. Employing a simplified real options model, it investigates the influence of three forms of flexibility, namely complete outsourcing, partial outsourcing, and outsourcing with flexibility to backsourse (re-insource), under two opposing regimes (with high and low capital-to-labor costs).

The paper makes four contributions. First, it shows that the probability of switching to the outsourcing option increases with higher demand volatility under high capital-to-labor costs, whereas the same probability decreases with higher demand volatility under low capital-to-labor costs. Second, it proves that partial outsourcing is not a viable choice in either regime. Third, contrary to the common perception that the value of flexibility increases with higher market volatility, it shows that the incremental benefits from the backsourcing flexibility can be decreasing with higher demand volatility. Fourth, it illustrates that backsourcing flexibility adds considerable value in only the high capital-to-labor costs regime and, as such, should only be a priority for firms operating under this regime.

Keywords: outsourcing, backsourcing, demand uncertainty, real options.

1. Introduction

We analyze how the tendency to outsource and timing of switches between sourcing modes are affected by the degree of demand uncertainty, by relative differences in sourcing costs, and by contract flexibility permitting the switches. We develop models for finding the optimal demand thresholds for outsourcing and backsourcing, the probabilities of actual demand reaching these thresholds, the cost savings expected from switching a sourcing mode, and the expected time to switching.

We consider a firm that is currently insourcing a transaction-based business process and that has the option to outsource the process. The operating cost structure is comprised of a variable cost per transaction and a fixed overhead cost per period. In the case of an outsourced process, the fixed cost per period is comprised of a per-period fee charged to the firm and/or the cost the firm accrues for monitoring the performance of the outside firm. Figure 1 illustrates the four possible relationships between the insourced and outsourced fixed costs as well as between the insourced and outsourced variable costs. The question of whether to outsource and when arises only when no sourcing mode offers an absolute cost advantage, as in the quadrants labeled Regime 1 and Regime 2 in Figure 1.
Figure 1: Outsourcing Regimes

Under Regime 1 the outsourced variable cost is higher than the insourced variable cost and the outsourced fixed cost is lower than the insourced fixed cost. Sample industries that fit this quadrant include: telemmedicine, software development, equity research, and credit research. These industries are characterized by: (1) high-cost, high-skill and low labor productivity, where less costly supply exists in other markets; (2) high-cost personnel training; and (3) high-cost administrative and infrastructure resources (e.g., IT assets, end-user support, real estate, etc.).

Under Regime 2 the outsourced variable cost is lower than the insourced variable cost and the outsourced fixed cost is higher than the insourced fixed cost. Sample industries fitting this quadrant include: call centers, medical transcription, claims processing, HR administration, and logistics or shipping. These industries are characterized by: (1) low-cost, low-skill and high labor productivity; (2) low-cost personnel training; (3) high-cost process governance (monitoring, quality assurance, error detection, reporting and handling); and (4) the variable costs are defined by set agreements on service-levels.

For each regime we examine three complementary scenarios. The first scenario is a binary case of complete outsourcing without the ability to break the outsourcing contract in midstream. We find that the effect of increasing demand volatility on the probability and timing of outsourcing depends critically on the cost structure regime. Under Regime 1 increased demand volatility generally increases (decreases) the probability (timing) of outsourcing. By contrast, under Regime 2 increased demand volatility generally decreases (increases) the probability (timing) of outsourcing.

The second and third scenarios introduce two forms of contract flexibility. The second scenario adds the possibility of partial outsourcing. Alvarez and Stenbacka (2007) consider an outsourcing model that conforms to Regime 2 (i.e., outsourced variable cost is lower than insourced variable cost) but with no fixed costs. The cost to switch to outsourcing exhibits diseconomies of scale with respect to the fraction of demand that is outsourced. Our models include fixed costs, and the switching cost is independent of the
fraction of demand outsourced (e.g., the cost of switching to the outsourcing alternative exhibits economies of scale). We show that, regardless of regime, it is never optimal to outsource a fraction of demand.

The third scenario adds the flexibility to backsource. The inclusion of contract flexibility never decreases the probability of outsourcing. Nevertheless, we find that the behaviors identified in the first scenario under both regimes continue to hold when backsourcing flexibility is introduced—the results are robust under additional contract flexibility. In addition, contrary to conventional wisdom, we find that the value of including the flexibility to backsource in the outsourcing contract can be decreasing in demand volatility under Regime 1. The result implies that it is less important for firms in highly volatile markets to build backsourcing flexibility into sourcing contracts than it is for firms in less volatile markets.

The paper contributes to the growing literature on sourcing decisions under environmental uncertainty (e.g., Van Mieghem 1999 and 2003, Kogut and Kulatilaka 1994, Kouvelis, Axarloglou and Sinha 2001, Alvarez and Stenbeack 2007, Lu and Van Mieghem, 2009). It presents a simple, and yet unique, analytical framework for investigating questions related to outsourcing. It also identifies a series of insights related to the value associated with the three flexibilities (outsourcing, partial outsourcing, and backsourcing), the likelihood of outsourcing, and the expected time to switch to outsourcing. Earlier studies typically focus on the value generated from the flexibility associated with the option to outsource. This paper departs from earlier publications as it also provides an analytical investigation regarding the likelihood of exercising the option to outsource, namely, the probability of switching to outsourcing and the expected time to making the switch.

Our paper makes a contribution also by offering four specific insights. First, we show that the influence of demand volatility on the probability of outsourcing depends on the operating characteristics. Specifically, under Regime 1 the probability of switching to outsourcing increases and the expected time to making the switch decreases with increasing demand volatility, whereas under Regime 2 the opposite behavior is observed. This result complements and enriches an earlier result according to which increased volatility widens the hysteresis band and suggests an increased persistence of the current strategy (Kogut and Kulatilaka 1994, Kouvelis et al. 2001). In these papers, wider hysteresis band is associated with organizational inertia where the firm resists a switch to a different strategy. We, too, find that the hysteresis band widens as volatility increases under both regimes, however, we also enrich this result by offering a formal description of persistence through the probability of switching to the outsourcing option. Specifically, we show that despite the widening hysteresis band, the probability of hitting the optimal threshold is also increasing in demand volatility under Regime 1, implying that the firm has a reduced persistence of the current sourcing strategy. Thus, higher demand volatility increases the hysteresis band, but the organizational inertia to deviate from the existing sourcing alternative depends on the operational setting, and indeed, it can be decreasing in demand volatility as is the case under Regime 1. Second, we show that
it is never beneficial for a firm to partially outsource its operations and send only a proportion of its demand volume to the outsource facility. Lu and Van Mieghem (2009) argue that incorporating fixed costs increases the likelihood of centralizing the production decisions either at the outsource or insource facility. Our finding formalizes their intuition in a different setting. Third, we show that, contrary to the common wisdom, the value gained from additional flexibility, in the form of a backsourcing capability, does not always increase with higher demand volatility. Van Mieghem (1999, 2003), Van Mieghem and Rudi (2003), Alvarez and Stenbeaka (2007) show that the value of outsourcing flexibility increases with higher demand volatility, and our findings are consistent with their results. Our paper extends their results by investigating the benefits of adding more flexibility in the form of a backsourcing capability. Unlike the outsourcing option, however, we show that the incremental value from the backsourcing flexibility does not always increase in demand volatility; indeed, it can be decreasing in demand volatility under Regime 1. Lastly, we illustrate that the value of backsourcing flexibility is higher under Regime 1 than under Regime 2.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the models of Regime 1 and Regime 2 under the first scenario with full outsourcing. Section 4 considers the impact of added flexibility in the form of partial outsourcing and the option to backsource. Section 5 concludes with a summary and suggestions for future research. All derivations are located in the Appendix.

2. Related Literature

Our work is related with four streams of research. The first deals with the use of real options in operational decision-making. Birge (2000) and Dixit and Pindyk (1994) provide good examples of how option pricing can incorporate financial risk attitudes into operational decision-making. Dixit (1989a, 1989b) demonstrate how these models can be used to determine whether a firm should enter and exit a foreign market. Similarly in this paper, we use real options analysis to capture the operational flexibility to outsource and backsourse a business process under demand uncertainty.

Using queuing systems, the second stream considers the firm’s subcontracting decisions of service operations, such as the use of call centers, under demand uncertainty. Ren and Zhou (2008) investigate how contracts can be used in terms of staffing and effort decisions in order to coordinate supply chains. Hasija, Pinker and Shumsky (2008) compare various payment methods in contracting, e.g. pay-per-time and pay-per-call, that lead to supply chain coordination under information asymmetry about staff productivity. Aksin, Vericourt and Karaesmen (2008) examine the firm’s subcontracting decision as to whether to outsource the base demand or the fluctuation of demand exceeding the base. Milner and Olsen (2008) rationalize a service provider’s decision to give priority to contracted customers over non-contracted cus-
tomers only when there is less traffic and not under heavy-traffic. These studies incorporate the costs of waiting and abandonment and focus on features of the payment methods that lead to coordination in supply chains. Considering scale economies, i.e., firm’s unit manufacturing cost decreases with higher demand, Cachon and Harker (2002) investigate pricing and outsourcing decisions under a queuing game competition. While these papers emphasize contracting decisions, we focus on the firm’s outsourcing and backsourcing decisions over time. Moreover, in terms of research methodology, we use option pricing theory rather than queuing theory.

The third stream of research deals with capacity decisions leading to production network design with multiple facilities under demand uncertainty. In these papers, the firm’s decision to invest in a secondary production facility resembles the outsourcing decision in our model. Van Mieghem (2003) provides a comprehensive summary of various capacity investment models. A majority of the models employed in this area are single-period models. Van Mieghem (1999) develops outsourcing conditions using price-only and incomplete contracts for subcontracting decisions. Van Mieghem and Rudi (2002) first establish a newsvendor network in a single-period model, and under identically and independently distributed market demands, show how the single-period optimal decisions stay in effect in a multiple period setting. Using a bivariate demand flow, Tan (2004) determines the firm’s optimal production and subcontracting decisions that ensure the long-term availability of capacity at the facility of the subcontractor. Tomlin and Wang (2005) compare the values generated from various forms of network design, specifically a single dedicated source for operations, two dedicated sources, single flexible source and two flexible sources. Similar to their study, Lu and Van Mieghem (2009) investigate the firm’s network design from a centralization versus decentralization (hybrid) perspective. Their optimal network design configuration depends on differences in prices, manufacturing and transportation costs between markets. While all of these studies consider primarily variable costs (of manufacturing and transportation), Lu and Van Mieghem (2009) mention the influence of fixed costs in the analysis and report that increasing fixed costs makes the centralization of operations in a single facility as the dominant strategy. Because these papers employ a single-period model, they do not capture the conditions for switching production activities between facilities. Adjustment of capacity over time is typically captured in multi-period models, however, the analyses in these studies are primarily restricted to the structure of the optimal policy. Our emphasis, on the other hand, provides insight into the probabilistic nature of outsourcing and backsourcing in a continuous-time model.

The fourth stream of research considers the influence of uncertainty in economic conditions such as the fluctuations in exchange rates in global production networks. Kogut and Kulatilaka (1994) is the first study to describe the flexibility to shift production between two manufacturing facilities with fluctuating exchange rates as equivalent to owning an option. They define market prices and the associated demand
as deterministic, but variables costs are influenced by exchange rate fluctuations. When fixed switching costs are incorporated, they find that the optimal policy structure features a hysteresis band where the firm does not shift production under smaller variations in exchange rates. In a two plant scenario, Dasu and Li (1997) extend this analysis using linear and step-function switching costs in a model where exchange rate uncertainty is described with a discrete-time Markov chain. They find that regardless of whether the variable costs are concave, piece-wise linear, or convex, the optimal policy structure features a barrier policy confirming the earlier description of a hysteresis band. Modeling exchange rate fluctuations with a stochastic diffusion process Huchzermeier and Cohen (1996) provide a comprehensive valuation approach to various network structures with multiple suppliers, production facilities and markets. They refine the definition of the option value of operational flexibility by incorporating the firm’s switching costs from one network structure to another. Kamrad and Lele (1999) also incorporate real options approach into characterizing optimal production policies under market price and supply uncertainty with less emphasis on switching costs. Kamrad and Siddique (2004) extend this work by incorporating supplier switching options and profit sharing and investigate their influence on supply contracts. Kouvelis, Axarloglou and Sinha (2001) study the influence of exchange rate uncertainty in the type of the ownership in production facilities in international markets (e.g. exporting, joint ventures, and wholly-owned subsidiaries). In their model, the fixed costs of switching increase as the firm’s ownership in the foreign subsidiary increases, i.e., the cost is higher for switches from exporting to wholly-owned subsidiary than to joint venture. They conclude that increasing switching costs extend the hysteresis band. In a single period model, Ding, Dong and Kouvelis (2007) determine the optimal production decisions along with the optimal choices for financial hedging instruments. Our paper differs from this stream of research in three ways. First, while the papers in this stream consider exclusively variable costs (i.e., ignoring fixed costs), our model incorporates economies of scale with the presence of fixed and variable costs. Second, production volume is a decision variable in these papers. In our study, however, the production volume is exogenous and the firm’s decision corresponds to the percentage of this volume to be outsourced. Third, the emphasis is on the structure of the optimal policy in this stream of research. In our study, we focus on the influence of demand volatility on the probability to outsource service operations.

Alvarez and Stenbacka (2007) provide the closest match to our model. They investigate the effect of increasing demand volatility on the expected time of a switch to outsourcing and on the fraction of volume to outsource. They consider the case where the outsourced cost per transaction is lower than the insourced cost per transaction, and they ignore fixed overhead costs. The cost to switch to outsourcing depends upon the fraction of volume that is outsourced. In particular, switching cost is increasing convex in the outsource fraction, thus exhibiting diseconomies of scale. They find that the firm will never partially
outsource when switching cost is linear in the fraction of volume outsourced. This model can be viewed as a special case of our model with zero fixed costs under Regime 2.

3. Full Outsourcing

A firm is currently insourcing a transaction-based business process and has the option to outsource the process at any time in the future. The process, if outsourced, must be completely outsourced (i.e., no partial outsourcing). In this section we consider two models that differ by operating cost structures. In Regime 1, the vendor’s variable cost per transaction is higher than the in-house variable cost per transaction but the outsourcing fixed cost per period is lower than the in-house fixed cost per period. The model and analyses of Regime 1 are given in Section 3.1.

In Regime 2, the opposite cost relationships hold; the vendor’s variable cost per transaction is lower than the in-house variable cost per transaction but the outsourcing fixed cost per period is higher than the in-house fixed cost per period. The model and analyses of Regime 2 are given is Section 3.2. Section 3.3 compares and contrasts the two regimes through a series of numerical experiments.

We outline the notation and the modeling elements that are common to both regimes below.

**Notation**

- $F_I = \text{fixed cost per period for a business process when insourced}$
- $F_O = \text{fixed cost per period for a business process when outsourced}$
- $v_I = \text{cost per process transaction when insourced}$
- $v_O = \text{cost per process transaction when outsourced}$
- $S_{IO} = \text{one-time fixed cost to outsource an existing insourced process}$
- $D_0 = \text{current transaction rate}$
- $D_t = \text{transaction rate at time } t$
- $r = \text{firm’s discount rate (net of inflation in firm and vendor operating costs)}$

**Assumptions**

- A1. The business process is currently insourced
- A2. Transaction demand over time is modeled as geometric Brownian motion: $dD_t = D_t\mu dt + D_t\sigma dz$
  where $\mu$ is the growth rate, $\sigma$ is the volatility, and $z(t)$ is a standard Weiner process with $z(0) = 0$
- A3. $r > \mu$

Assumption A2 is common in the literature and implies that the change in transaction volume over time conforms to a lognormal distribution. Assumption A3 is the standard absence of speculative bubbles condition.

When the process is insourced, the cost rate is

$$c_I(D_t) = v_ID_t + F_I,$$
when the process is outsourced, the cost rate is
\[ c_O(D_t) = v_0D_t + F_O, \]
and the expected total discounted cost of insourcing over an infinite horizon is
\[ C_I(D_0) = E \left[ \int_0^\infty e^{-r\tau} (v_0D_t + F_I) dt \right]. \]

### 3.1 Regime 1 – Outsource at Higher Variable Cost and Lower Fixed Cost

Under Regime 1 we have the following additional assumption.

A4. \( v_I < v_O \) and \( F_I > F_O \)

Due to A4, \( c_O(D_t) \leq c_I(D_t) \) if and only if \( D_t \leq \frac{v_O - v_I}{F_I - F_O} \), i.e., low demand rates favor outsourcing and high demand rates favor insourcing. Suppose the firm elects to switch to outsourcing if and when the demand rate hits a threshold rate \( D < D_0 \). Such a switching policy is optimal due to the stationarity of \( dz \). Let \( \tau_1(D) \) denote the random time of the switch to outsourcing, i.e.,
\[ \tau_1(D) = \min\{ t \mid D_t \leq D \}. \]
The firm’s expected discounted operating cost under Regime 1 is
\[ C_1(D_0, D) = E \left[ \int_0^\infty e^{-r\tau} (v_0D_t + F_I) dt + \int_{\tau_1(D)}^\infty e^{-r\tau} (v_0D_t + F_O) dt + e^{-r\tau_1(D)} S_{IO} \right] \]
\[ = C_I(D_0) - V_1(D) \quad (1) \]
where
\[ V_1(D) = \frac{D}{D_0} \left[ \left( \frac{F_I - F_O - rS_{IO}}{r} \right) - \left( \frac{v_O - v_I}{r - \mu} \right) D \right] \quad (2) \]
\[ \gamma = \sqrt{\left( \mu - 0.5\sigma^2 \right)^2 + 2\sigma^2 r + \left( \mu - 0.5\sigma^2 \right)} \frac{1}{\sigma^2}. \]
The function \( V_1(D) \) is the value of the option to outsource given that the firm switches to outsourcing when the current demand rate passes below (or hits) threshold \( D \). If \( F_I - F_O - rS_{IO} \leq 0 \), then it is apparent from (2) that the firm will never outsource (i.e., if \( F_I - F_O - rS_{IO} \leq 0 \), then \( V_1(D) \leq 0 \) for all \( D > 0 \)). Thus, for the remainder of this section we limit consideration to cases where \( F_I - F_O - rS_{IO} > 0 \). The optimal threshold is
\[ D^* = \arg\max_{D \geq 0} V_1(D) = \frac{\gamma (r - \mu) (F_I - F_O - rS_{IO})}{r (\gamma + 1) (v_O - v_I)}. \quad (3) \]
The value of the option to outsource is obtained by substituting (3) into (2) while accounting for the fact that it is optimal for the firm to immediately switch to outsourcing if \( D_0 \leq D^* \).
We let $\tau_1^*$ denote the optimal random time that the firm switches to outsourcing (i.e., $\tau_1^* = \tau_1(D^*)$).

The probability of outsourcing is

$$P[\tau_1^* < \infty] = \begin{cases} \left(\frac{D^*}{D_0}\right)^{\frac{2\mu}{\sigma^2}} & \text{if } \mu - 0.5\sigma^2 \geq 0 \text{ and } D_0 \geq D^* \\ 1 & \text{if } \mu - 0.5\sigma^2 \leq 0 \text{ or } D_0 \leq D^* \end{cases}$$

and the expected time to switch to the outsourcing alternative is

$$E[\tau_1^*] = \begin{cases} \infty & \text{if } \mu - 0.5\sigma^2 \geq 0 \text{ and } D_0 \geq D^* \\ -\left(\frac{1}{\mu - 0.5\sigma^2}\right) \ln \left(\frac{D_0}{D^*}\right) & \text{if } \mu - 0.5\sigma^2 \leq 0 \text{ and } D_0 \geq D^* \\ 0 & \text{if } D_0 \leq D^* \end{cases}$$

### 3.2 Regime 2 – Outsource at Lower Variable Cost and Higher Fixed Cost

Under Regime 2 we have the following assumption in place of A4.

A5. $v_I < v_O$ and $F_I \geq F_O$

Due to A5, $c_O(D_t) \leq c_I(D_t)$ if and only if $D_t \geq \frac{v_I - v_O}{F_O - F_I}$, i.e., high demand rates favor outsourcing and low demand rates favor insourcing. In the event of $F_I = F_O$, we have $c_O(D_t) < c_I(D_t)$, and the decision to switch to outsourcing involves a trade-off between the outsourcing variable cost savings and the switching cost.

Suppose the firm elects to switch to outsourcing if and when the demand rate hits a threshold rate $D > D_0$. Such a switching policy is optimal due to the stationarity of $dz$. Let $\tau_2(D)$ denote the random time of the switch to outsourcing, i.e.,

$$\tau_2(D) = \min\{t \mid D_t \geq D\}.$$

The firm’s expected discounted operating cost under Regime 2 is

$$C_2(D_0, D) = E \left[ \int_0^{\tau_2(D)} e^{-\rho t} (v_I D_t + F_I) \, dt + \int_{\tau_2(D)}^{\infty} e^{-\rho t} (v_O D_t + F_O) \, dt + e^{-\rho \tau_2(D)} S_{\theta_0} \right].$$

$$= C_I(D_0) - V_2(D).$$

where

$$V_2(D) = \left(\frac{D_0}{D}\right)^{\frac{\rho}{\gamma}} \left[ \left(\frac{v_I - v_O}{r - \mu}\right) D - \left(\frac{F_O - F_I + r S_{\theta_0}}{r}\right) \right]$$

$$= \left(\frac{D_0}{D}\right)^{\frac{\rho}{\gamma}} \left[ \left(\frac{v_I - v_O}{r - \mu}\right) D - \left(\frac{F_O - F_I + r S_{\theta_0}}{r}\right) \right]$$
\[ \beta = \sqrt{\left(\mu - 0.5\sigma^2\right)^2 + 2\sigma^2 r \left(\mu - 0.5\sigma^2\right)} / \sigma^2. \]

The function \( V_2(D) \) is the value of the option to outsource given that the firm switches to outsourcing when the current demand rate passes above (or hits) threshold \( D \). The optimal threshold is

\[ D^{**} = \arg \max_{D \geq 0} V_2(D) = \begin{cases} 
\frac{\beta(r - \mu)(F_0 - F_i + rS_{i0})}{r(\beta - 1)(v_i - v_o)} & \text{if } \beta > 1, \\
\infty & \text{if } \beta \leq 1.
\end{cases} \quad (9) \]

If \( \beta \leq 1 \), then it is apparent from (9) that the firm will never outsource. Thus, for the remainder of this section we limit consideration to cases where \( \beta > 1 \).

The value of the option to outsource is obtained by substituting (9) into (8) while accounting for the fact that it is optimal for the firm to immediately switch to outsourcing if \( D_0 \geq D^{**} \).

\[ V_2^* = \begin{cases} 
\left( \frac{D_0(v_i - v_o)}{\beta(r - \mu)} \right)^{\frac{\beta}{\beta - 1}} \frac{r(\beta - 1)}{F_0 - F_i + rS_{i0}} & \text{if } D_0 \leq D^{**}, \\
\left( \frac{v_i - v_o}{r - \mu} \right) D_0 - \frac{F_0 - F_i + rS_{i0}}{r} & \text{if } D_0 > D^{**}
\end{cases} \quad (10) \]

We let \( \tau_2^* \) denote the optimal random time that the firm switches to outsourcing (i.e., \( \tau_2^* = \tau_2(D^{**}) \)).

The probability of outsourcing is

\[ P[\tau_2^* < \infty] = \begin{cases} 
\left( \frac{D^{**}}{D_0} \right)^{\frac{2\beta - 1}{\beta}} & \text{if } \mu - 0.5\sigma^2 \leq 0 \text{ and } D_0 \leq D^{**}, \\
1 & \text{if } \mu - 0.5\sigma^2 > 0 \text{ or } D_0 > D^{**}
\end{cases} \quad (11) \]

and the expected time to switch to the outsourcing alternative is

\[ E[\tau_2^*] = \begin{cases} 
\infty & \text{if } \mu - 0.5\sigma^2 \leq 0 \text{ and } D_0 \leq D^{**}, \\
\frac{1}{\mu - 0.5\sigma^2} \ln \left( \frac{D^{**}}{D_0} \right) & \text{if } \mu - 0.5\sigma^2 > 0 \text{ and } D_0 \leq D^{**}, \\
0 & \text{if } D_0 > D^{**}
\end{cases} \quad (12) \]

We note that the expected time to outsourcing is finite when the probability of outsourcing is 100%, and is infinite otherwise. Thus, both measures are needed to provide a complete picture of the effect of changing parameters values on outsourcing, as illustrated in the next section. The probability of outsourcing, for example, is not included in Alvarez and Stenbacka (2007) who consider a model that is similar to our model under Regime 2.

### 3.3 Numerical Analysis

In this section we report the effect of changing environmental conditions on the likelihood, timing, and value of outsourcing under both regimes. Figures 2 and 3 illustrate the effect of changing demand volatili-
ty, variable cost differential, fixed cost differential, and demand growth rate on four measures: the value of the option to outsource ($V_1^*$ in Regime 1, and $V_2^*$ in Regime 2), the optimal switching demand threshold ($D^*$), the probability of outsourcing ($P[t_t^* < \infty]$), and expected time to outsourcing ($E[t_t^*]$). Table 1 shows the range of parameter values used in the figures.

Table 1: Parameter Values in Figures 2 and 3

<table>
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<tr>
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<th>Regime 1</th>
<th>Regime 2</th>
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<td>900</td>
</tr>
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<td>10%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>6%</td>
<td>2%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4% - 40%</td>
<td>4% - 40%</td>
</tr>
<tr>
<td>$v_I$</td>
<td>$0.10$</td>
<td>$0.14$</td>
</tr>
<tr>
<td>$v_O$</td>
<td>$0.14$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>$F_I$</td>
<td>$300$</td>
<td>$200$</td>
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<tr>
<td>$F_O$</td>
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<td>$300$</td>
</tr>
<tr>
<td>$S_{IO}$</td>
<td>$15$</td>
<td>$15$</td>
</tr>
</tbody>
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Under Regime 1, where outsourcing has a higher variable cost but a lower fixed cost than the insourcing alternative, the firm would prefer to utilize outsourcing over insourcing when the demand for the service process is low. Figure 2 depicts the influence of problem parameters, primarily the volatility in demand, on the outsourcing decision under Regime 1. First, it can be seen that the demand threshold ($D^*$) for switching to outsourcing decreases in demand volatility under all conditions. The demand threshold also decreases when the difference in variable costs increases or when the difference in fixed costs increases. Increasing values of the demand growth rate also reduces this demand threshold. Second, consistent with the findings in the real options theory literature, the value of the option to outsource increases in demand volatility. Moreover, the value of this option increases with: 1) the decreasing values of the difference in variable costs, 2) the increasing values of the difference in fixed costs, and 3) the decreasing values of the demand growth rate. Thus, the behavior of this option value is similar to the demand threshold.

While a monotone behavior is observed for the demand threshold and the value of the outsourcing option in Figure 2, the probability of switching to the outsourcing alternative and the expected time to take this action do not follow a monotone behavior with the demand volatility. For some problem parameters, the probability of switching to the outsourcing alternative can be 100% for relatively small demand volatility (e.g., the firm immediately switches to outsourcing) and for relatively high demand volatility. However, for intermediate demand volatility values, the probability can exhibit both an increasing and decreasing behavior. Overall, large differences in variable costs as well as increased demand growth rates cause the probability of switching to the outsourcing alternative to be close to zero under relatively lower demand volatility, and the probability increases with the demand volatility. By contrast, smaller differ-
ences in fixed costs tend to increase the probability of switching. However, what is consistent is that higher demand volatility causes the probability to approach 100%. That is, significant demand volatility increases the probability of outsourcing despite the fact it also decreases the demand threshold.

![Graphs](image)

(a) Variable cost differential \((v_O - v_I)\)

\((v_O\) range is $0.12 - $0.2; \(\mu=1\%)\)

(b) Fixed cost differential \((F_I - F_O)\)

\((F_I\) range is $230 - $360)

(c) Demand growth rate \((\mu)\)

\((\mu\) range is 0% - 8%)

Figure 2: Analysis of Regime 1

While not monotonic, the expected time to switch to the outsourcing alternative decreases with increasing demand volatility. The firm prefers the outsourcing alternative earlier in time under significantly high demand volatility. The expected time for the switching decision is generally postponed with the increasing values of the difference in the variable costs, with the decreasing values of the fixed costs, and with increasing values of the growth rate.

Under Regime 2, where outsourcing has a higher fixed cost but a lower variable cost, the firm would prefer to utilize the outsourcing alternative over the insourcing alternative when demand for the process is high. Figure 3 illustrates the influence of problem parameters on the outsourcing decisions for Regime 2. It can be observed that the demand threshold \((D^{**})\) for switching to outsourcing increases in demand volatility under all conditions. The increase in demand threshold is smaller when the difference in variable costs increases, the difference in fixed costs decreases, or the demand growth rate increases; these conditions make it more likely that the firm will make the switch to outsourcing. Once again, consistent with the findings of the real options theory literature, the value of the option to outsource increases in demand volatility. The behavior of this option value exhibits an opposite behavior than the demand threshold.
Specifically, the value of the outsourcing option increases as: 1) the difference in variable costs increases, 2) the difference in fixed costs decreases, and 3) the demand growth rate increases.

Unlike in Regime 1, the probability of switching to the outsourcing alternative shows a monotone behavior under Regime 2. (We note, however, that a non-monotone behavior—increasing then decreasing—can arise when the demand growth rate is negative.) For low demand volatility values, the probability of switching to the outsourcing alternative is highest, and is closest to 100% in most problem instances. However, as demand volatility increases, the probability of switching to the outsourcing alternative decreases. And, once the probability starts decreasing, it continues to decrease in the demand volatility. Thus, higher demand volatility implies that insourcing is a better alternative for the firm. In general, though, increasing values of the difference in variable costs, or increasing growth rates in demand, amplify the attractiveness of the outsourcing alternative, increasing the probability of outsourcing. Decreasing values of the difference in the fixed costs have the same effect on the probability of switching to the outsourcing alternative.

The behavior of the expected time to switch to the outsourcing alternative under Regime 2 also departs from that under Regime 1. An increase in the demand volatility implies an increase in the expected time to switch. Thus, the firm would postpone the switching decision with increasing demand volatility.
Moreover, under significant demand volatility, the expected time to switch to the outsourcing alternative becomes unbounded (i.e., when the probability of outsourcing is less than 100%). Similar to the observations made for the probability of switching to the outsourcing alternative, the expected time to making the switch decreases with increasing values of the difference in the variable costs, with decreasing values of the difference in the fixed costs, and with increasing values of the demand growth rate. Under these conditions, the firm would prefer to switch to the outsourcing alternative earlier in time.

4. Impact and Value of Additional Contract Flexibility

In this section we extend our models to account for two forms of outsourcing contract flexibility. In Section 4.1 we consider the case where the firm is free to outsource any given fraction of transaction (demand) volume. In Section 4.2 we study the setting where the firm has the option to break the outsourcing contract and bring the process back in-house (e.g., backsourcing flexibility). Section 4.3 contains numerical analysis of outsourcing with backsourcing flexibility under the two regimes.

4.1 Partial Outsourcing Flexibility

A firm, which is currently insourcing a business process, is free to outsource any fraction \( p \in (0, 1] \) of transaction volume at any time in the future. Section 3 considered the special case of \( p = 1 \). When fraction \( p \) of demand is outsourced, the insource cost rate is

\[
c_I(D, p) = v_I D(1 - p) + F_I(1 - p)
\]

and the outsource cost rate is

\[
c_O(D, p) = v_O D p + F_O p^\eta
\]

where \( \eta \) is the outsourcing fixed cost scale parameter. We assume \( \eta \in (0, 1] \). The assumption implies that the fixed cost per period when demand is split between two facilities is not less than the weighted average of \( F_I \) and \( F_O \)—the overhead costs when volume is entirely insourced or entirely outsourced, i.e.,

\[
F_I(1 - p) + F_O p^\eta \geq F_I(1 - p) + F_O p \quad \text{for any } p \in (0, 1]\.
\]

The case of \( \eta < 1 \) reflects real-world settings where diseconomies of scale arise when demand is split between two firms.

We begin with Regime 1. Suppose the firm elects to outsource some fraction \( p \in (0, 1] \) of demand if and when the demand rate hits a threshold rate \( D < D_0 \). The firm’s expected discounted operating cost is

\[
C_1(D_0, D, p) = C_I(D_0) - p \left( \frac{D}{D_0} \right)^\gamma \left( \frac{F_I - F_O p^\eta - v_S \mu}{r} \right) + \left( \frac{v_O - v_I}{r - \mu} \right) D \] \tag{13}

which is strictly decreasing in \( p \). Consequently, the firm will either choose to fully outsource or never outsource. The firm will never partially outsource demand.
We find the same result under Regime 2. Given threshold $D > D_0$, the firm’s expected discounted operating cost is

$$C_2(D_0, D, p) = C_1(D_0) - p \left( \frac{D_0}{D} \right)^{\eta} \left[ \left( \frac{v_I - v_O}{r - \mu} \right) D - \left( \frac{F_O p^{\eta-1} - F_I + rS_{IO}}{r} \right) \right],$$

which is strictly decreasing in $p$. Consequently, the firm will either choose to fully outsource or never outsource. The firm will never partially outsource demand.

In summary, the flexibility to partially outsource adds no value. The conclusion is in contrast with a result in Alvarez and Stenbacka (2007) who find that partial outsourcing can add value. Their model is similar to our model under Regime 2 except that fixed costs at the in-house facility and the outsource facility are assumed to be zero, or equivalently, the fixed costs are identical. A key difference is that they assume that switching cost is convex in the fraction outsourced, i.e., $S_{IO} = k + (K/b)p^b$ where $b > 1$. The convexity manifested in $b > 1$ puts downward pressure on the value of $p$, which is not present in our model. In fact, partial outsourcing is never optimal in their model if $b = 1$.

### 4.2 Outsourcing with Backsourcing Flexibility

Backsourcing flexibility can be valuable in a variety of service settings. For example, SAP INFO Solutions indicates that 20-30% of all BPO contracts are terminated within two years and 80% need to be renegotiated (Mani et al., 2005; Bloch and Spang, 2003), and more broadly about half of all IT outsourcing contracts are terminated or renegotiated within the first year (Weakland, 2006; D’Agostino, 2006; Bartholemy, 2001). In most such cases, services are either brought back in-house or outsourced to another vendor. Similar reports are starting to appear for offshored manufacturing services as well (Holstein, 2010). In this light, this section examines the impact of having the flexibility to backsource on the initial decision to outsource.

A firm, which is currently insourcing a business process, has the option to outsource at any time in the future. In addition, the firm has the option to backsource (i.e., switch to in-house processing) at any time in the future. The cost to include this provision in the outsourcing contract is denoted $S_{OI}$. Alternatively, the value of $S_{OI}$ can be interpreted as the cost of breaking the outsourcing contract.

#### 4.2.1 Outsourcing with Backsourcing Flexibility Under Regime 1

Under Regime 1 we have $v_I < v_O$ and $F_I > F_O$, and thus low demand rates favor outsourcing and high demand rates favor insourcing. Suppose the firm elects to switch to outsourcing if and when the demand rate hits a threshold rate $D < D_0$. As in Section 3.1, we let $\tau_I(D)$ denote the random time of the switch to outsourcing, i.e.,

$$\tau_I(D) = \min \{ t \mid D_t \leq D \}.$$
At the moment of the switch to outsourcing, the firm has the option to switch back to insourcing at any
time in the future. The optimal threshold for switching back to insourcing once a firm has switched to
outsourcing is

\[ D^+ = \begin{cases} 
\beta(r - \mu)(F_i - F_o + r S_{ol}) & \text{if } \beta > 1 \\
\infty & \text{if } \beta \leq 1 
\end{cases} \]

where

\[ \beta = \sqrt{\left(\mu - 0.5\sigma^2\right)^2 + 2\sigma^2 r - \left(\mu - 0.5\sigma^2\right)} \]

From (15) we see that if \( \beta \leq 1 \), then the firm would never backsource after the firm has switched to out-
sourcing. In the remainder of this section, we limit consideration to cases where \( \beta > 1 \).

The value of the backsourcing option at the moment the firm switches to outsourcing (i.e., the mo-
moment in time when the demand rate hits threshold \( D \)) is

\[ V_2^*(D) = \begin{cases} 
\left(\frac{D(v_o - v_i)}{\beta(r - \mu)}\right)^\beta \left(\frac{r(\beta - 1)}{F_i - F_o + r S_{ol}}\right)^{\beta-1} & \text{if } D \leq D^+ \\
\left(v_o - v_i\right)D - \left(\frac{F_i - F_o + r S_{ol}}{r}\right) & \text{if } D \geq D^+ 
\end{cases} \]

The firm’s expected discounted operating cost is the same as in Section 3.1 except that the cost to switch
from insourcing to outsourcing (\( S_{io} \)) is reduced by the value of the backsourcing option (\( V_2^*(D) \)) that is
activated at the moment the firm switches to outsourcing, i.e.,

\[ C_i(D_o, D) = E \left[ \int_0^{\gamma(D)} e^{-\alpha} (v_i D_i + F_i) dt + \int_{\gamma(D)}^{\infty} e^{-\alpha} (v_o D_i + F_o) dt + e^{-r_0(D)} (S_{io} - V_2^*(D)) \right] \]

\[ = C_i(D_o) - V_3(D) \]

where

\[ V_3(D) = \left(\frac{D}{D_o}\right)^\gamma \left[\left(\frac{F_i - F_o - r(S_{io} - V_2^*(D))}{r}\right) - \left(v_o - v_i\right)\right] \]

\[ \gamma = \sqrt{\left(\mu - 0.5\sigma^2\right)^2 + 2\sigma^2 r - \left(\mu - 0.5\sigma^2\right)} \]

The optimal threshold is

\[ D^o = \arg \max_{D \in [D_0, D^+]} V_3(D) \]

where, due to the constraints,
There is no closed form expression for \( D^o \) (i.e., \( D^o \) is the root of a high order polynomial). However, \( D^o \) can be obtained numerically using readily available software such as Nonlinear Solver in Microsoft Excel.

The probability that the firm will switch to outsourcing and the expected time to switch to the outsourcing alternative can be obtained from expressions (5) and (6). Given that the firm switches to outsourcing, the probability that the firm will switch back to insourcing and the expected time to switch to the insourcing alternative can be obtained from expressions (11) and (12) (\( D^{++} \) is used in place of \( D^* \) and \( D^o \) is used in place of \( D_0 \)).

### 4.2.2 Outsourcing with Backsourcing Flexibility Under Regime 2

Under Regime 2 we have \( v_I > v_O \) and \( F_I \leq F_O \), and thus high demand rates favor outsourcing and low demand rates favor insourcing. Suppose the firm elects to switch to outsourcing if and when the demand rate hits a threshold rate \( D > D_0 \). As in Section 3.2, we let \( \tau_2(D) \) denote the random time of the switch to outsourcing, i.e.,

\[
\tau_2(D) = \min\{t \mid D_t \geq D\}.
\]

At the moment of the switch to outsourcing, the firm has the option to switch back to insourcing at any time in the future.

A firm that has switched to outsourcing will never choose to backsource if \( F_O - F_I - rS_{IO} \leq 0 \) (see (2)). Thus, for the remainder of this section we limit consideration to cases where \( F_I - F_O - rS_{IO} > 0 \). The optimal threshold for switching back insourcing once a firm has switched to outsourcing is

\[
D^u = \frac{\gamma(r - \mu)(F_o - F_i - rS_{IO})}{r(\gamma + 1)(v_i - v_o)}
\]

where

\[
\gamma = \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2r + (\mu - 0.5\sigma^2)}.
\]

The value of the backsourcing option at the moment the firm switches to outsourcing (i.e., the moment in time when the demand rate hits threshold \( D \)) is

\[
V_1^*(D) = \begin{cases} 
\left(\frac{F_O - F_I - rS_{IO}}{r}\right) - \left(\frac{v_i - v_o}{r - \mu}\right)D & \text{if } D \leq D^u \\
\left(\frac{\gamma(r - \mu)}{D(v_i - v_o)}\right)^{\gamma + 1} \left(\frac{F_O - F_I - rS_{IO}}{r(\gamma + 1)}\right)^{\gamma + 1} & \text{if } D \geq D^u
\end{cases}
\]
The firm’s expected discounted operating cost is the same as in Section 3.2 except that the cost to switch from insourcing to outsourcing \( (S_{IO}) \) is reduced by the value of the backsourcing option \( (V_1^*(D)) \) that is activated at the moment the firm switches to outsourcing, i.e.,

\[
C_2(D_0, D) = E \left[ \int_{0}^{r_2(D)} e^{-rt} \left( v_1 D_t + F_t \right) dt + \int_{r_2(D)}^{\infty} e^{-rt} \left( v_0 D_t + F_0 \right) dt + e^{-r \tau} \left( S_{IO} - V_1^* (D) \right) \right] \\
= C_1(D_0) - V_4(D)
\]

where

\[
V_4(D) = \left( \frac{D_0}{D} \right)^{\beta} \left[ \left( \frac{v_1 - v_0}{r - \mu} \right) D - \left( \frac{F_0 - F_1 + r \left( S_{IO} - V_1^*(D) \right) }{r} \right) \right]
\]

\[
\beta = \sqrt{\left( \mu - 0.5 \sigma^2 \right)^2 + 2 \sigma^2 r - (\mu - 0.5 \sigma^2)^2}.
\]

The optimal threshold is

\[
D^{\infty} = \arg \max_{D \geq D_0, D > D^*} V_4(D)
\]

where, due to the constraints,

\[
V_1^*(D) = \left( \frac{\gamma (r - \mu)}{D (v_1 - v_0)} \right)^{\gamma} \left( \frac{F_0 - F_1 - r S_{IO}}{r (\gamma + 1)} \right)^{\gamma + 1}.
\]

There is no closed form expression for \( D^{\infty} \) (i.e., \( D^{\infty} \) is the root of a high order polynomial). However, \( D^{\infty} \) can be obtained numerically using readily available software such as Nonlinear Solver in Microsoft Excel.

The probability that the firm will switch to outsourcing and the expected time to switch to the outsourcing alternative can be obtained from expressions (11) and (12). Given that the firm switches to outsourcing, the probability that the firm will switch back to insourcing and the expected time to switch to the insourcing alternative can be obtained from expressions (5) and (6) in Section 3.1 (\( D^* \) is used in place of \( D^* \) and \( D^{\infty} \) is used in place of \( D_0 \)).

### 4.3 Numerical Analysis

Figures 4 and 5 illustrate the effect of changing volatility, variable cost differential, fixed cost differential, and demand growth rate on four measures: the optimal demand threshold \( (D^*) \) for switching to outsourcing, the probability of outsourcing \( (P[t_1^* \leq \infty]) \), the expected time to outsourcing \( (E[t_1^*]) \), and the incremental value of the backsourcing option \( (V_3(D^*) - V_1^* \text{ in Regime 1, and } V_4(D^{\infty}) - V_2^* \text{ in Regime 2}) \). Table 2 shows the range of parameter values used in the figures.
Table 2: Parameter Values in Figures 4 and 5

<table>
<thead>
<tr>
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<th>Regime 1</th>
<th>Regime 2</th>
</tr>
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<td>900</td>
</tr>
<tr>
<td>$r$</td>
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<td>$\mu$</td>
<td>6%</td>
<td>2%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4% - 40%</td>
<td>4% - 40%</td>
</tr>
<tr>
<td>$v_I$</td>
<td>$0.10$</td>
<td>$0.14$</td>
</tr>
<tr>
<td>$v_O$</td>
<td>$0.14$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>$F_I$</td>
<td>$300$</td>
<td>$200$</td>
</tr>
<tr>
<td>$F_O$</td>
<td>$200$</td>
<td>$300$</td>
</tr>
<tr>
<td>$S_{IO}$</td>
<td>$15$</td>
<td>$15$</td>
</tr>
<tr>
<td>$S_{OI}$</td>
<td>$20$</td>
<td>$20$</td>
</tr>
</tbody>
</table>

Figure 4 illustrates the influence of the backsourcing option under various problem parameters under Regime 1. The incremental value of the backsourcing option, given by $V_3(D^o) - V_1^*$, generally exhibits an increasing pattern with a reversed bowl-shape for some problem parameters. It increases for relatively low values of the demand volatility, but decreases or just converges to a relatively stable value for higher demand volatilities. The latter occurs for lower values of the difference in variable costs, lower values of the demand growth rate, and higher values of the difference in fixed costs. To understand the basis for this pattern, it is important to revisit the value of a “plain” option to outsource with no backsourcing flexibility (see Figure 2). When the value of the plain option to outsource is small (e.g., for low values of the demand volatility and high (low) values of the difference in variable (fixed) costs), the net value contribution of backsourcing is relatively small; an exception is for higher demand growth rates. When the value of the plain option to outsource is relatively large (e.g., for higher values of the demand volatility and lower (higher) values of the difference in variable (fixed) costs), the net value contribution of backsourcing is large but declining for higher demand volatilities; again, an exception is for higher demand growth rates.

That the incremental value of the backsourcing flexibility decreases with increased demand volatility can be explained using the properties of the model. As the demand volatility increases in our model, the stochastic demand process described by the geometric Brownian motion can hit the absorbent state (zero) with a higher probability. As the probability of experiencing zero demand increases, implying that the business is no more viable, the backsourcing option becomes less valuable, and its net value contribution starts decreasing with increasing volatility.

It is necessary to point out that earlier literature provides examples of how increasing market volatility creates a higher value for various forms of flexibility (Van Mighem 1999, 2003, Van Mieghem and Rudi 2002, Alvarez and Stenbecka 2007). Consistent with the findings in the real options theory literature, the common perception is that the value of flexibility increases with higher volatility. Yet, the above result regarding the incremental value of backsourcing flexibility departs from this common wisdom.
With this said, however, under Regime 1 the backsourcing flexibility still has a significant beneficial effect on the decision to switch to outsourcing. Specifically, when compared with Figure 2, incorporating the backsourcing option makes the outsourcing alternative more desirable. The demand threshold is consistently higher than for a plain option to outsource (Figure 2). And, while the demand threshold continues to decrease monotonically under all problem parameters, there is a point where the decrease gets tapered off significantly with increasing values of the demand volatility. As a result, the probability of switching to the outsourcing alternative increases under all problem parameters, and the expected time to making the switch goes to zero when the probability is close enough to 100%. Once again, though, the behavior of the probability is not monotone in the demand volatility. For certain problem parameter values, the probability increases for low values of the demand volatility, then become wave-shaped or just U-shaped for intermediate values of demand volatility, and eventually becomes 100% for high demand volatilities. This is exactly the case seen in Figure 4 for decreasing values of the difference in the variable costs, increasing values of the difference in the fixed costs, and increasing values of the demand growth rate. And, again, when the probability drops enough below 100%, the expected time to making the switch to outsourcing becomes excessively large.

In summary, we conclude that the presence of backsourcing flexibility in Regime 1 is beneficial under all circumstances, but its incremental value can get diminished under higher demand volatilities.
Figure 5 illustrates the influence of the backsourcing option for various problem parameters under Regime 2. Two observations can be made regarding the incremental value of the backsourcing flexibility. First, the net value contribution of backsourcing flexibility is significant only when the value of the plain option to outsource (with no backsourcing) is large; otherwise it is small or close to zero. Second, the net value contribution of the backsourcing flexibility is significantly less when the demand volatility is low under all problem parameters, and it increases with higher demand volatility. Thus, it can be concluded that the contribution of backsourcing flexibility can be significant only under relatively high values of the demand volatility. It is necessary to point out that while the incremental value of the backsourcing flexibility can be decreasing with higher volatility in Regime 1, it is strictly increasing in Regime 2. However, apparent from the other results in Figure 5, this does not necessarily make outsourcing more likely.

With regard to the optimal demand threshold under Regime 2, while it exhibits a continuous increasing behavior when no backsourcing flexibility exists (see Figure 3), the presence of backsourcing flexibility generally lowers the demand threshold, especially for high values of demand volatility. However, the drop in the optimal demand threshold is not enough to increase the probability of outsourcing close to 100%. Specifically, the probability of outsourcing grows only marginally for intermediate and high values of the demand volatility and for all other problem parameters. Similarly, no beneficial impact can be ob-
served for the expected time to switch to the outsourcing alternative. Thus, the inclusion of the backsourcing flexibility has a marginal impact on the probability of outsourcing and the expected time to outsource.

It is important to point out that the net value contribution of the backsourcing flexibility is significantly higher under Regime 1 than in Regime 2. A firm operating under the cost structure of Regime 2 will have less to gain from inclusion of this form of flexibility. This is due to the long-term positive growth under both regimes. Expected demand is increasing in time, and thus in the long term (i.e., under high volumes), the firm is better off processing transactions in-house under Regime 1 and outsourcing under Regime 2. In growth markets, the flexibility to reduce cost under Regime 2 by bringing transaction processing back in-house adds relatively little value.

In summary, we conclude that the introduction of backsourcing flexibility 1) adds value in both regimes, but its contribution is significantly more beneficial under Regime 1; 2) increases the probability of outsourcing; 3) reduces the expected time to outsourcing in both regimes; and 4) the influence of problem parameters on the probability of outsourcing and the expected time to outsource for each regime continue to hold.

5. Summary and Conclusions

We define and analyze models for sourcing a business process under two opposing cost structures. The cost structures correspond to the two cases wherein the decision of whether to outsource is nontrivial.

Under Regime 1 the outsourced variable cost is higher than the insourced variable cost and the outsourced fixed cost is lower than the insourced fixed cost. Regime 1 is typical of high capital-to-labor processes such as IT-based business processes. IT-based processes are characterized by relatively low labor content and relatively high investment in information infrastructure. Fixed operating costs incurred by the outsource firm are generally incorporated into the price charged per transaction.

Under Regime 2 the outsourced variable cost is lower than the insourced variable cost and the outsourced fixed cost is higher than (or the same as) the insourced fixed cost. Regime 2 is typically observed in business environments that have low capital-to-labor processes. Examples of this infrastructure are observed in call centers and garment production. The firm may be charged a low variable cost by the outsource firm, but incurs higher fixed costs (relative to in-house processing) in the form of monitoring and quality assurance expenses.

The results of our study lead to four main conclusions. First, the impact of demand volatility on the likelihood and timing of a switch to outsourcing depends critically on the regime. Under Regime 1, as demand volatility increases, the probability of outsourcing generally increases and the expected time to a switch to outsourcing generally decreases. Under Regime 2 we find the opposite behavior; as demand volatility increases the probability of outsourcing generally decreases and the expected time to a switch to
outsourcing generally increases. Thus, as economic conditions become more volatile, our models predict greater levels of outsourcing in high capital-to-labor processes and lower levels of outsourcing in low capital-to-labor processes. We find that these predictions continue to hold when the flexibility to back-source (i.e., to break the contract in midstream) is introduced.

This conclusion complements and enriches an earlier result on the relationship between volatility and the hysteresis band in a different setting. Kogut and Kulatilaka (1994) and Kouvelis, Axarloglou and Sinha (2001), for example, examine the impact of exchange rate volatility on ownership structures and find that the hysteresis band is increasing in exchange rate volatility. In these studies, the increasing hysteresis band is associated with increasing persistence of the current strategy, or in other words, increasing organizational inertia. We, too, find that the hysteresis band is increasing in volatility under both regimes, e.g., the optimal demand threshold decreases in volatility under Regime 1 and increases in volatility under Regime 2. However, we also find that under Regime 1 the probability of hitting the optimal threshold defining the hysteresis band is increasing in volatility, indicating that organizational inertia is actually decreasing in demand volatility.

A second conclusion is that, from a cost perspective, it does not make sense to partially outsource a process. Expected cost is lower (or possibly the same) if the firm, upon a decision to outsource, outsources 100% of the volume instead of some fraction of the volume. This contrasts with the result of Alvarez and Stenbacka (2007) and holds true even in the case where fixed costs are prorated according to volume (i.e., no diseconomies of scale when splitting volume between two facilities).

The third conclusion is that there is a relationship between volatility and the value of flexibility which differs from conventional wisdom and earlier results. Earlier research points out that the higher the demand volatility the higher the value of flexibility (Van Mieghem 1999 and 2003, Van Mieghem and Rudi 2002, Alvarez and Stenbacka 2007, and Lu and Van Mieghem 2009). We show that the incremental value from incorporating the backsourcing flexibility can be decreasing with higher demand volatility. This result occurs under Regime 1 where businesses operate with high capital-to-labor processes. In this environment, higher demand volatility increases the likelihood of outsourcing and, at the same time, increases the likelihood that the firm will go out of business. The increasing probability of liquidation mitigates the benefit of backsourcing flexibility, even to the point where the value of backsourcing flexibility begins to decrease.

The last conclusion is that the beneficial effect of backsourcing flexibility is more significant under Regime 1 than Regime 2. In this light, the inclusion of backsourcing flexibility should be higher priority for firms with high capital-to-labor processes.

Our models are stylized representations of reality that rely on two key assumptions—assumptions that may limit the applicability of our conclusions. First, we assume that demand conforms to geometric
Brownian motion. If the demand process hits zero, it stays at zero (e.g., the firm goes out of business). The presence of a single absorbing state contributes to the regime-dependent effect of demand volatility on the probability of outsourcing. As demand volatility increases, the probability of hitting the absorbing state increases, which in turn increases the attractiveness of the sourcing option that offers lower cost at low volumes. Under Regime 1, the sourcing option favored at low volumes is outsourcing, and the probability of outsourcing is increasing in demand volatility. Under Regime 2, the sourcing option favored at low volumes is insourcing, and the probability of outsourcing is decreasing in demand volatility. Second, we assume an infinite horizon and stable parameter values over the horizon. Under a brief problem horizon with non-stable parameters over time, the conclusions may change. Investigating the influence of changes in these assumptions and an empirical evaluation of our predictions are worthy topics for future research.

Appendix

Derivation of (1)

\[
C_i(D_0, D) = E \left[ \int_0^{\tau(D)} e^{-r t} \left( v_i D_t + F_i \right) dt + \int_{\tau(D)}^{\infty} e^{-r t} \left( v_o D_t + F_o \right) dt + e^{-r \tau(D) S_{\infty}} \right]
\]

\[
= E \left[ \int_0^{\tau(D)} e^{-r t} \left( v_i D_t + F_i \right) dt \right] - E \left[ \int_{\tau(D)}^{\infty} e^{-r t} \left[ (F_i - F_o) - (v_o - v_i) D_t \right] dt - e^{-r \tau(D) S_{\infty}} \right]
\]

\[
= C_i(D_0) - E \left[ e^{-r \tau(D)} \left( \int_0^{\tau(D)} e^{-r t} \left[ (F_i - F_o) - (v_o - v_i) D_t \right] dt - S_{\infty} \right) \right]
\]

The remaining steps take advantage of three identities.

\[
E[D_t] = D_0 e^{rt}
\]

(A1)

\[
\int_0^{\tau(D)} e^{-rt} \left( ae^{rt} + b \right) dt = \frac{a}{r - \mu} + \frac{b}{r} \text{ for } r > \mu
\]

(A2)

\[
E \left[ e^{-r \tau(D)} \right] = \left( \frac{D}{D_0} \right)^\gamma \text{ for } D < D_0
\]

(A3)

where

\[
\gamma = \sqrt{\left( \mu - 0.5 \sigma^2 \right)^2 + 2 \sigma^2 r \left( \mu - 0.5 \sigma^2 \right)}.
\]

(A1) can be obtained by taking the expectation of a geometric Brownian motion process. (A2) can be obtained by standard integration rules. (A3) can be obtained from a Laplace transform involving the density of \( \tau(D) \). Substituting identities (A1) – (A3) into \( C_i(D_0, D) \) yields

\[
C_i(D_0, D) = C_i(D_0) - \left( \frac{D}{D_0} \right)^\gamma \left( \frac{F_i - F_o - r S_{\infty}}{r} \right) - \left( \frac{v_o - v_i}{r - \mu} \right) D.
\]
= C_i(D_0) - V_1(D).

**Derivation of (3)**
Taking the derivative of $V_1(D)$, we have

$$V_1'(D) = \frac{D^{\gamma-1}}{D_0^\gamma} \left[ \gamma \left( \frac{F_i - F_o - rS_{io}}{r} \right) - (\gamma + 1) \left( \frac{v_o - v_i}{r - \mu} \right) D \right].$$

We see that $V_1(D)$ has a unique positive stationary point $D^* = \frac{\gamma (r - \mu) (F_i - F_o - rS_{io})}{r (\gamma + 1) (v_o - v_i)}$. Furthermore, $V_1'(D) \geq 0$ for all $D \in [0, D^*)$ and $V_1'(D) < 0$ for all $D \in (D^*, \infty)$. Thus $V_1(D)$ is quasi-concave over $D \in [0, \infty)$ and is maximized at $D^*$.

**Derivation of (4)**
Note that if $D_0 \leq D^*$, then the current demand rate is at or below the threshold where it is optimal to switch to outsourcing. Thus the firm immediately switches to outsourcing and the value of outsourcing is

$$V_1^* = \left( \frac{F_i - F_o - rS_{io}}{r} \right) - \left( \frac{v_o - v_i}{r - \mu} \right) D_0.$$

In the event of $D_0 \geq D^*$, we substitute

$$D^* = \arg \max_{D \geq 0} V_1(D) = \frac{\gamma (r - \mu) (F_i - F_o - rS_{io})}{r (\gamma + 1) (v_o - v_i)}$$

into $V_1(D)$ to get

$$V_1^* = \frac{D^*}{D_0} \left[ \left( \frac{F_i - F_o - rS_{io}}{r} \right) - \left( \frac{v_o - v_i}{r - \mu} \right) D^* \right] = \left( \frac{\gamma (r - \mu)}{D_o (v_o - v_i)} \right)^{\gamma \gamma} \left( \frac{F_i - F_o - rS_{io}}{r (\gamma + 1)} \right)^{\gamma + 1}.$$

**Derivation of (5)**
For geometric Brownian motion process $dD_t = D_t \mu dt + D_t \sigma dz$, random demand at time $t$ is $D_t = D_0 e^{(\mu - 0.5 \sigma^2)t + \sigma z(t)}$ where $z(t)$ is a standard Weiner process. Dixit (2001) gives the probability that an arithmetic Brownian motion process will hit a lower threshold, which can be used in conjunction with the following relationship to obtain (5).

$$P[\min\{D_t\} \leq D^*] = P[\min\{X_t\} \leq \ln(D^*/D_0)]$$

where $X_t = (\mu - 0.5 \sigma^2) t + \sigma z(t)$ is arithmetic Brownian motion.

**Derivation of (6)**
Dixit (2001) gives the expected time that an arithmetic Brownian motion process will hit a lower threshold, which can be used via the transformation employed in the derivation of (5) to obtain (6).

**Derivation of (13)**

$$C(D_0, p) = E \left[ \int_0^{\tau(D)} e^{-r_t} (v_i D_t + F_i) dt + \int_{\tau(D)}^{\infty} e^{-r_t} \left[ (1 - p)(v_i D_t + F_i) + p(v_o D_t + F_o p^{g-1}) \right] dt + e^{-r(D)} S_{io} \right]$$

$$= C_i(D_0) - p E \left[ \int_{\tau(D)}^{\infty} e^{-r_t} \left[ (F_i - F_o p^{g-1}) - (v_o - v_i) D_t \right] dt - e^{-r(D)} S_{io} \right]$$
\[ C_i(D_0) - pE \left[ e^{-rt(D)} \left( \int_0^t e^{-\nu} \left[ (F_i - F_o p^{v-1}) - (v_0 - v_i) \right] dt - S_{\theta} \right) \right] \]

\[ = C_i(D_0) - p \left( \frac{D}{D_0} \right) \left[ \frac{F_i - F_o p^{v-1} - rS_{\theta}}{r} - \frac{(v_0 - v_i)}{r - \mu} D \right] \]

**Derivation of (15)**
The derivation of (15) follows the derivation (9) that gives the optimal threshold for switching to a lower variable cost (and higher fixed cost), except that \( F_i, F_o, \) and \( S_{\theta} \) are replaced with \( F_o, F_i, \) and \( S_{\theta} \) respectively.

**Derivation of (16)**
Section 4.1 shows how to determine the value of the option to switch from a higher variable cost and a lower fixed cost (e.g., insourcing in Section 4.1, but outsourcing in Section 3.3) to a lower variable cost and a higher fixed cost (e.g., outsourcing in Section 4.1, but insourcing in Section 3.3). The value of the option to insource is obtained from equation (10) but the cost rates interchanged and initial demand rate \( D \), i.e.,

\[ V_2^*(D) = \begin{cases} \left( \frac{D(v_0 - v_i)}{\beta(r - \mu)} \right)^{\beta} \left( \frac{r(\beta - 1)}{F_i - F_o + rS_{\theta}} \right)^{\beta-1} & \text{if } D \leq D^+ \\ \left( \frac{v_0 - v_i}{r - \mu} \right) D - \left( \frac{F_i - F_o + rS_{\theta}}{r} \right) & \text{if } D \geq D^+ \end{cases} \]

where

\[ \beta = \sqrt{\left( \mu - 0.5\sigma^2 \right)^2 + 2\sigma^2 r - (\mu - 0.5\sigma^2)} \]

**Derivation of (17)**
In addition to \( D \geq 0 \), the optimization problem

\[ D^* = \arg \max_{D \geq 0, D \leq D_0, D < D^+} V_3(D) \]

has two other constraints: \( D \leq D_0 \) and \( D < D^+ \). If \( D > D_0 \), then the firm immediately switches to outsourcing, and thus \( D_0 \) represents an upper limit on the threshold to switch to outsourcing (i.e., \( D \leq D_0 \)). An outsource-to-insource threshold value satisfying \( D \geq D^+ \) is not viable because such a threshold implies that the moment the demand rate hits \( D \), the firm switches from insourcing to outsourcing, then immediately back to insourcing, thus incurring two switching costs with no change in the operation. Thus, \( D < D^+ \) and expression (16) becomes

\[ V_2^*(D) = \left( \frac{D(v_0 - v_i)}{\beta(r - \mu)} \right)^{\beta} \left( \frac{r(\beta - 1)}{F_i - F_o + rS_{\theta}} \right)^{\beta-1} \]

As an aside,

\[ V_3(D) = \left( \frac{D}{D_0} \right)^\gamma \left[ \left( \frac{F_i - F_o - rS_{\theta}}{r} \right) - \left( \frac{v_0 - v_i}{r - \mu} \right) D \right] + \left( \frac{D}{D_0} \right)^\gamma V_2^*(D) \]

The first term in brackets is quasi-concave in \( D \) (see derivation of (3)) and, given \( \beta > 1 \) (which implies \( \beta + \gamma > 2 \)), the second term in brackets is convex in \( D \). Thus, \( V_3(D) \) is not necessarily quasi-concave (which complicates the characterization of \( D^* \)).

**Derivation of (7)**
We make use of the following identity (Shackelton and Wójcikowski 2002).
\[ E \left[ e^{r \tau_2(D)} \right] = \left( \frac{D}{\bar{D}} \right)^\beta \quad \text{for } D > D_0 \]  

where  
\[ \beta = \frac{\sqrt{\left( \mu - 0.5\sigma^2 \right)^2 + 2\sigma^2 r - \left( \mu - 0.5\sigma^2 \right)}}{\sigma^2}. \]

Accordingly,  
\[
C_2(D_0, D) = E \left[ \int_{\tau_2(D)} e^{-rt} \left( v_i D_i + F_i \right) dt + \int_{\tau_2(D)} e^{-rt} \left( v_o D_i + F_o \right) dt + e^{-r\tau_2(D)} S_{\infty} \right]
\]
\[
= E \left[ \int_{0}^{\infty} e^{-r\tau} \left( v_i D_i + F_i \right) dt \right] - E \left[ \int_{\tau_2(D)}^{\infty} e^{-r\tau} \left( v_i - v_o \right) D_i \left( F_o - F_i \right) dt - e^{-r\tau_2(D)} S_{\infty} \right]
\]
\[
= C_i(D_0) - E \left[ e^{-r\tau_2(D)} \left( \int_{\tau_2(D)}^{\infty} e^{-r\tau} \left[ (v_i - v_o) D_i \left( F_o - F_i \right) \right] dt - S_{\infty} \right] \right]
\]

Substituting identities (A1) – (A4) into \( C_2(D_0, D) \) yields  
\[
C_2(D_0, D) = C_i(D_0) - \left( \frac{D_0}{D} \right)^\beta \left[ \left( \frac{v_i - v_o}{r - \mu} \right) D - \left( \frac{F_o - F_i + rS_{\infty}}{r} \right) \right].
\]

Derivation of (9)  
Taking the derivative of \( V_2(D) \), we have  
\[
V_2'(D) = \frac{D_0}{D^{\alpha+1}} \left[ \beta \left( \frac{F_o - F_i + rS_{\infty}}{r} \right) - (\beta - 1) \left( \frac{v_i - v_o}{r - \mu} \right) D \right].
\]

If \( \beta \leq 1 \), then \( V_2'(D) > 0 \) for all \( D \geq 0 \), and the optimal threshold is \( D^* = \infty \). If \( \beta > 1 \), then \( V_2(D) \) has a unique positive stationary point \( D^* = \frac{\beta (r - \mu) (F_o - F_i + rS_{\infty})}{r (\beta - 1) (v_i - v_o)} \). Furthermore, \( V_2'(D) > 0 \) for all \( D \in (0, D^*) \) and \( V_2'(D) < 0 \) for all \( D \in (D^*, \infty) \). Thus \( V_2(D) \) is quasi-concave over \( D \in [0, \infty) \) and is maximized at \( D^* \).

Derivation of (10)  
Note that if \( D_0 \geq D^* \), then the current demand rate is at or above the threshold where it is optimal to switch to outsourcing. Thus the firm immediately switches to outsourcing and the value of outsourcing is  
\[
V_2^* = \left( \frac{v_i - v_o}{r - \mu} \right) D_0 - \left( \frac{F_o - F_i + rS_{\infty}}{r} \right).
\]

In the event of \( D_0 \leq D^* \), we substitute  
\[
D^* = \arg\max_{D>0} V_2(D) = \frac{\beta (r - \mu) (F_o - F_i + rS_{\infty})}{r (\beta - 1) (v_i - v_o)}
\]

into \( V_2(D) \) to get  
\[
V_2^* = \left( \frac{D_0}{D^*} \right)^\beta \left[ \left( \frac{v_i - v_o}{r - \mu} \right) D^* - \left( \frac{F_o - F_i + rS_{\infty}}{r} \right) \right].
\]
Derivation of (11)
Dixit (2001) gives the probability that an arithmetic Brownian motion process will hit an upper threshold, which can be used to obtain (11) (see Derivation of (5)).

Derivation of (12)
Dixit (2001) gives the expected time that an arithmetic Brownian motion process will hit an upper threshold, which can be used to obtain (12) (see Derivation of (6)).

Derivation of (14)

\[
C_2(D_0, D, p) = E \left[ \int_0^{\tau_{(D)}} e^{-\alpha} (v_D + F_D) dt + \int_{\tau_{(D)}}^\infty e^{-\alpha} \left( (1 - p)(v_D + F_D) + p(v_D + F_D p^{\theta-1}) \right) dt + e^{-\alpha \tau_{(D)}} S_{IO} \right]
\]

\[
= C_1(D_0) - pE \left[ \int_0^{\tau_{(D)}} e^{-\alpha} \left( (v_D - v_o) D_D - \left( F_D p^{\theta-1} - F_I \right) \right) dt + e^{-\alpha \tau_{(D)}} S_{IO} \right]
\]

\[
= C_1(D_0) - pE \left[ e^{-\alpha \tau_{(D)}} \left( \int_0^{\tau_{(D)}} e^{-\alpha} \left( (v_D - v_o) D_D - \left( F_D p^{\theta-1} - F_I \right) \right) dt + S_{IO} \right) \right]
\]

\[
= C_1(D_0) - p \left( \frac{D_0}{D} \right)^\beta \left[ \left( \frac{v_D - v_o}{r - \mu} \right) D - \left( \frac{F_D p^{\theta-1} - F_I + r S_{IO}}{r} \right) \right]
\]

Derivation of (18)
The derivation of (18) follows the derivation (3) that gives the optimal threshold for switching to a higher variable cost (and lower fixed cost), except that \( FI, FO, \) and \( SI_O \) are replaced with \( FO, FI, \) and \( SO_I \) respectively.

Derivation of (19)
Section 3.1 shows how to determine the value of the option to switch from a lower variable cost and a higher fixed cost (e.g., insourcing in Section 3.1, but outsourcing here) to a higher variable cost and a lower fixed cost (e.g., outsourcing in Section 3.1, but insourcing here). The value of the option to insource is obtained from equation (4) but the cost rates interchanged and initial demand rate \( D \), i.e.,

\[
V_1^*(D) = \begin{cases} 
\left( \frac{F_o - F_i - r S_{oi}}{r} \right) - \left( \frac{v_i - v_o}{r - \mu} \right) D & \text{if } D \leq D^* \\
\frac{r (r - \mu)}{D(v_i - v_o)} \left( \frac{F_o - F_i - r S_{oi}}{r (\gamma + 1)} \right)^{\gamma + 1} & \text{if } D \geq D^*
\end{cases}
\]

where

\[
\gamma = \sqrt{ \left( \mu - 0.5 \sigma^2 \right)^2 + 2 \sigma^2 r + \left( \mu - 0.5 \sigma^2 \right)}
\]

Derivation of (20)
The optimization problem

\[
D^{so} = \arg \max_{D \leq D_0, D > D^*} V_4(D)
\]

has two constraints: \( D \geq D_0 \) and \( D > D^* \). If \( D < D_0 \), then the firm immediately switches to outsourcing, and thus \( D_0 \) represents a lower limit on the threshold to switch to outsourcing (i.e., \( D \geq D_0 \)). An outsource-
to-insource threshold value satisfying \( D \leq D^* \) is not viable because such a threshold implies that the moment the demand rate hits \( D \), the firm switches from insourcing to outsourcing, then immediately back to insourcing, thus incurring two switching costs with no change in the operation). Thus, \( D > D^* \) and expression (19) becomes

\[
V_1^*(D) = \left( \frac{\gamma (r - \mu)}{D(v_r - v_o)} - \left( \frac{F_o - F_r - rS_{ol}}{r(y + 1)} \right)^{\gamma+1} \right).
\]

As an aside,

\[
V_4(D) = \left( \frac{D_o}{D} \right)^\beta \left[ \left( r - \mu \right) \left( v_r - v_o \right) D - \left( \frac{F_o - F_r + rS_{ol}}{r} \right) \right] + \left( \frac{D_o}{D} \right)^\gamma V_1^*(D).
\]

Given \( \beta > 1 \), the first term in brackets is quasi-concave in \( D \) (see derivation of (9)) and the second term in brackets is convex in \( D \). Thus, \( V_4(D) \) is not necessarily quasi-concave (which complicates the characterization of \( D^{oo} \)).

**References**


